PHY 484 Week 2:
Review of Special Relativity

4-Vectors:

(Inertial frames)

$$x^\mu = (ct, \vec{x})$$

with Lorentz Transformation

$$\Lambda^\mu_{\ \nu} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

[in $\hat{x}$-direction]

then $x'^\mu = \Lambda^\mu_{\ \nu} x^\nu$ (Einstein summation).

Lorentz invariance: any Lorentz scaler.

covariant

contravariant

$$x^\mu x_\mu = x^\mu g_{\mu\nu} x^\nu$$

= invariant in any frame

with $g_{\mu\nu} = (1, -1, -1, -1)$ signature

metric tensor

A 4-vector (interval) $a^\mu = \begin{cases} \text{timelike} & a^2 > 0 \\ \text{spacelike} & a^2 < 0 \\ \text{lightlike} & a^2 = 0 \end{cases}$ (separation)

Proper measurements:

- $d\tau = \frac{dt}{\gamma}$ time measured in rest frame of object

  "Proper time"

- $\vec{\eta} = \frac{d\vec{x}}{d\tau}$ is then "proper velocity" (4-vector)

  $\eta^\mu = (\eta^0, \vec{\eta}) = (\gamma c, \frac{d\vec{x}}{d\tau})$

- $p^\mu = (\frac{E}{c}, \vec{p})$ "Energy-momentum 4-vector"

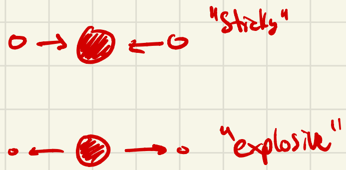
Massless / Massive momentum:

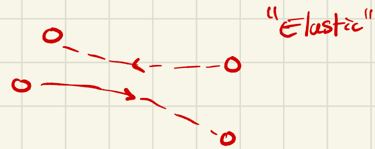
- $E = \gamma mc^2$ (massive particles only)

- For photons: deBroglie wavelength $\lambda = \frac{h}{p} \Longrightarrow E = h\nu = \frac{hc}{\lambda} = |\vec{p}|c.$

Collisions (Generic form) : $A + B \to C + D$

- Classically, mass / momentum / (maybe) Energy must be conserved.





- Relativistically, Energy/momentum are <u>always</u> conserved

  Kinetic energy / mass <u>may or may not</u> be conserved.

  i) "Sticky" : mass $\uparrow$ KE $\downarrow$

  ii) "Explosive" : mass $\downarrow$ KE $\uparrow$

  iii) "Elastic" : mass, KE conserved.

- Invariant Mass : a system of N particles has an invariant mass defined by $M_{inv}^2 c^2 = P_{TOT} \cdot P_{TOT}$

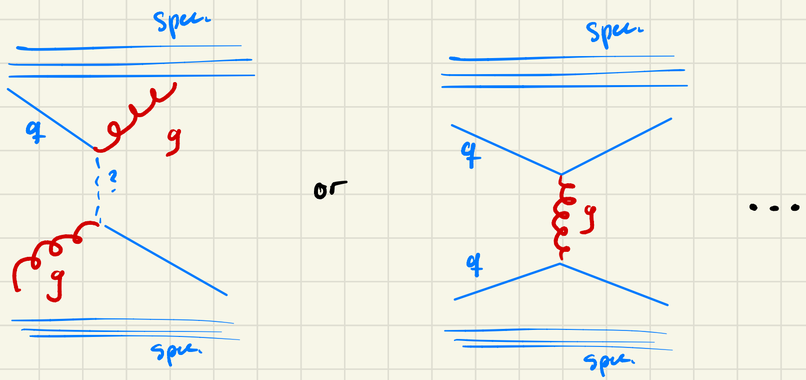
  with $P_{TOT} = \sum_{i=1}^{N} P_i$.

  Then $M_{inv}^2 c^2 = P_{TOT}^{\mu} P_{TOT\,\mu} = \frac{E_{TOT}^2}{c^2} - |\vec{P}_{TOT}|^2$.

LHC pp Collisions:

- High energy (14 TeV) proton collisions – really scattering between constituents of protons (quarks, g – "PARTONS")

ie.



- Square of COM energy $S = (P_1 + P_2)^2$

At high energies, very small – $P_1^2 = P_2^2 = m_p^2 c^2$

Square of COM energy (proton-proton)

$$= P_1^2 + P_2^2 + 2 P_1 \cdot P_2$$

$$\simeq 2 P_1 \cdot P_2$$

For parton-parton system, assume that the constituents involved in the fundamental collision each carry only a fraction the total proton momentum: fractions x_1, x_2. Then

$$\hat{S} = (x_1 P_1 + x_2 P_2)^2$$

square of COM energy (parton-parton)

$$\simeq 2 x_1 x_2 \, P_1 \cdot P_2$$

represents the square of the effective CoM energy of the parton-parton collision.

- The average parton-parton collision energy:

  estimate the fraction of proton energy carried by individual partons (take peak values for quarks and assume qq, qg and gg collisions are equally probable):

$$\hat{x}_q = \frac{1}{3}(0.2) + \frac{1}{3}(0.2) + \frac{1}{3}(0.1) = 0.17$$

$$\hat{x}_g \simeq 0.08$$

then $\langle \hat{x}_q \hat{x}_g \rangle = 0.013$

$$\langle \hat{s} \rangle = 0.013\, s \Rightarrow \hat{E}_{CM} \sim \sqrt{0.013}\, E_{CM} = (0.11)(14 \text{ TeV})$$

$$= 1.5 \text{ TeV}.$$